Theory of Automata

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1 Question 1

An input-output process is to simulate a coin-tossing game. The game is played by tossing a coin and scoring points according to the following rules: The first Head of a run of Heads scored two points, but the remaining Heads in the run do not score at all. In a run of tails, each odd-numbered Tail (the first, third, fifth, etc.) scores one point, and each even numbered Tail does not score any. For example, the sequence THHHTTTTTHTHH of tosses results in the scores 1200101012120, where T and H denote Tail and Head, respectively. All problems like the following lead eventually to an equation in that simple form.

1. Can the process be combinational, or must it be sequential? **Answer:** Since the process must use MEMORY to keep track of the previous states (first, second, third..), its current state not only depends on input but also on previos states so it must be SEQUENTIAL.

2. Can we construct a transducer to do the above computation? It yes, show it. Otherwise, explain why. **Answer:** we CANNOT construct transducer to do the above computation because of keep tracking the odd-numbered Tail (in other word to to memorize the number of each increment). For HEADs score, it's feasible since the transducer only has to remember the previous state.

2 Question 2

Use induction on n to show that $|u^n| = n|u|$ for all string u and n. We have a definition as:

1. $|a| = 1$

2. $|wa| = |w| + 1$ for all $a \in \sum$ and w any string on $\sum$

   As proven in the class we also have: $|uv| = |u| + |v|$.

   Base case: when $n = 1$ we have $|u^1| = |u|$ so it's true for the base case.

   Induction step:

   Assume that $|u^n| = n|u|$ is TRUE for all $n = k$. We need to show that $|u^n| = n|u|$ is TRUE for all $n = k + 1$.

   $|u^{k+1}| = |u * u^k| = |u| + |u^k|$. Since $|u^k| = k|u|$ is TRUE.

   $|u| + |u^k| = |u| + k|u| = (k + 1)|u|$.

   By induction step $|u^n| = n|u|$ is TRUE.

3 Question 3

The reverse of a string, introduced informally above, can be defined more precisely by the recursive rules

- $a^R = a$
- $(wa)^R = aw^R$ for all $a \in \sum$ and $w \in \sum^*$

Use this point to prove that: $(uv)^R = v^R u^R$, for all $u, v \in \sum^+$

Prove by induction: Because $u, v \in \sum^+$ so their length are not ZERO so we can use induction technique.

Assume that $(uv)^R = v^R u^R$ is TRUE for $|v| = n$.

We need to show that $(uv)^R = v^R u^R$ is TRUE for $|v| = n + 1$. Say that $v = wa$ where $|w| = n$

Left side: $(uv)^R = (uwa)^R = a(uw)^R = aw^R u^R$ (because of the assumption).

$aw^R u^R = (wa)^R u^R = v^R u^R$. This equals to the right side.

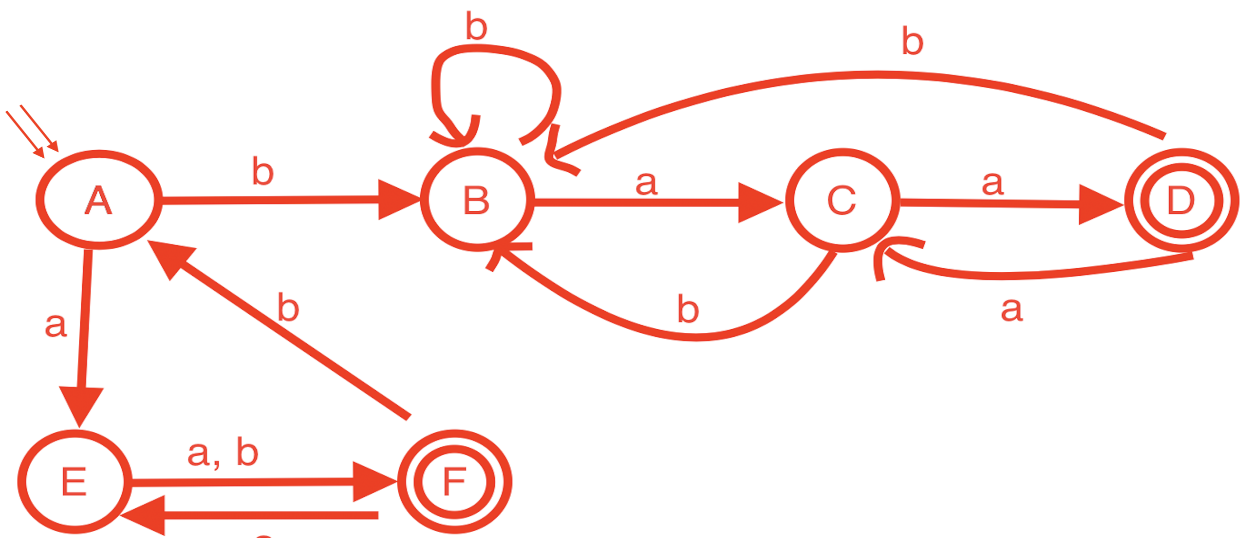
4 Question 4

Let $L = \{ab, aa, baa\}$. Which of the following strings are in L^*:

$abaabaaabaa, aaaabaaaa, baaaaabaaaab, baaaaabaa$?

First attempt, we can decompose the strings into substrings

- $abaabaaabaa = (ab)(aa)(baa)(ab)(aa)$ in L^*
- $aaaabaaaa = (aa)(aa)(baa)(aa)$ in L^*
- $baaaaabaaaab = (baa)(aa)(aa)(ab)(aa)(aa)b$ NOT in L^*
- $baaaaabaa = (baa)(aa)(ab)(aa)$ in L^*.

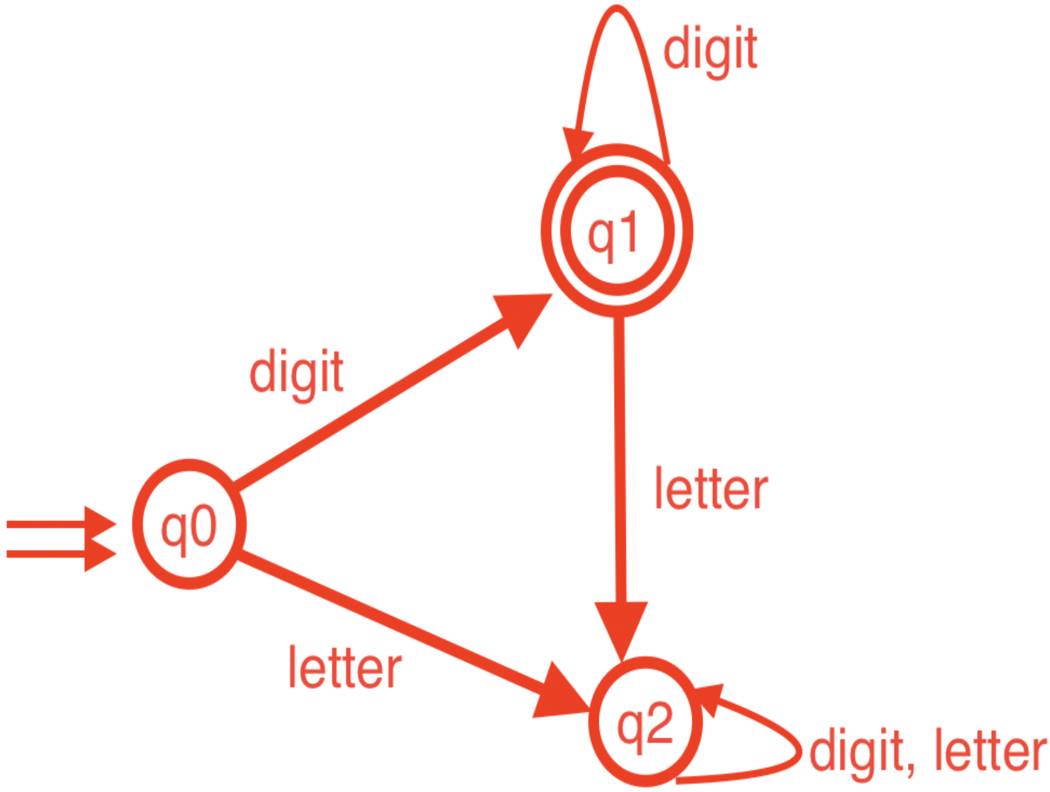


ch

Second attempt. Build a DFA then test the string.

5 Question 5

Design an acceptor for integers in C and Pascal. The solution is shown in Figure below.



6 Question 6

Let $a_1 a_2 ...$ be an input bit string. Design a transducer that computes the parity of every sub-string of three bits. Specifically, the transducer should produce output.

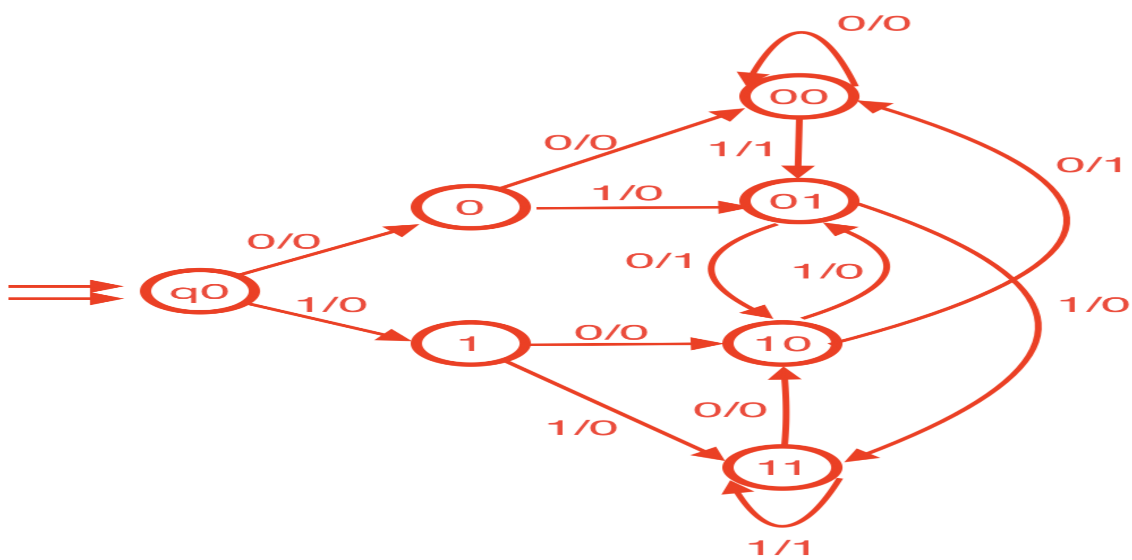
$\pi_1 = \pi_2 = 0$

$\pi_i = (a_{i-2} + a_{i-1} + a_i)$mode 2, $i = 3, 4, ...$

For example, the input 110111 should produce 000001.

Answer:

Since the final state only cares about the previous two bits. The output will be based on current input and two previous bits. We can label current state as ab where $a, b \in \{0, 1\}$, so we have 4 processing states where the transducer begins calculating. $00, 01, 10, 11$. To track back to the initial states when transducer reads the first two bits, we need 3 states. It is noted that for every three bits abc, the output is 1 when the number of 1 is odd, and the output is 0 when the number of 1 is even.



7 Question 7

Design a transducer - Moore machine - that calculates residues mod 5 for binary string input.

Answer: The machine is constructed and is shown in Figure below

input	design value	output
0	0	0
1	1	1
10	2	2
11	3	3
100	4	4
101	5	0
110	6	1
111	7	2
1000	8	3
1001	9	4
1010	10	0

(a)

q0: 5m	0	2*5m	q0
	1	2*5m + 1	q1
q1: 5m +1	0	2 * (5m+1)	q2
	1	2 * (5m+1) +1	q3
q2: 5m +2	0	2 * (5m+2)	q4
	1	2 * (5m+2) +1	q0
q3: 5m +3	0	2 * (5m+3)	q1
	1	2 * (5m+3) +1	q2
q4: 5m +4	0	2 * (5m+4)	q3
	1	2 * (5m+4) +1	q4

(b)